CHAPTER NO:- —— **UNIT – VIII** (1)

Bending Moments and Shearing forces

1. INTRODUCTION

When any structure loaded, the stresses are induced in the various parts of the structure either it may be tensile or compressive in nature. To calculate the induced stresses, where the structure is supported at a number of points, the bending moments and shearing forces acting must be calculated. In general, a structure is a combination of beams linked together in some way, and further, the complete structure may be treated as a beam with an elaborate cross section.

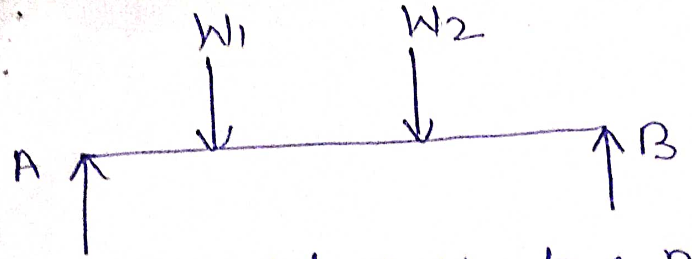
2. BEAM :-

A beam is a horizontal member of any structure which ~~having~~ have transverse loading i.e loads acts perpendicular to the axis of the member. For example member of rail bridge, building etc.

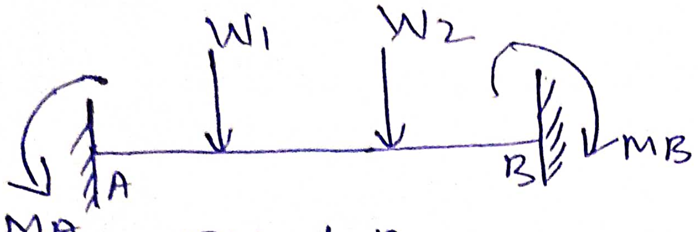
2.1 Types of Beams :- Beams are classified according to the supporting systems i.e types of supports and no of supports. These may be classified as following :-

2.1.1 Simply supported Beam :- A beam is simply supported if it rests freely at the supports. It is free to rotate about supports. The bending moment at the supports is always zero. See fig No-1

2.1.2 FIXED BEAM :- A fixed beam is one whose both ends are rigidly fixed at the ~~end~~ supports or built in into its supporting wall or column. The ends are not free to rotate about support. Fixing moments exerted by the support



Simply Supported Beam Fig-1



Fixed Beam Fig 2

on the beam i.e MA and MB are the fixing moments.
See fig No. 2

2.1.3 Cantilever Beam :-

A beam is said to be cantilever if its ~~supported~~ one end is fixed rigidly in the supporting wall or column. One end of the beam is free to deflect and to rotate. Fixing moments act at fixed support.

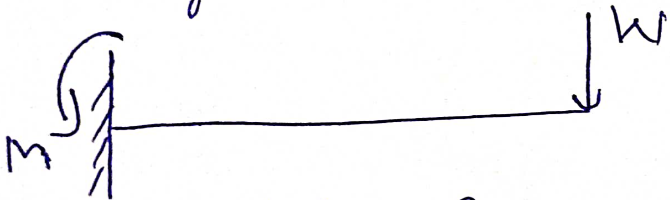


Fig-3

Cantilever Beam

2.1.4 Overhanging Beam :-

An over-hanging beam is one ~~of~~ if its ends extends over the support. The extended or projected portion of the beam over the support is called overhang.

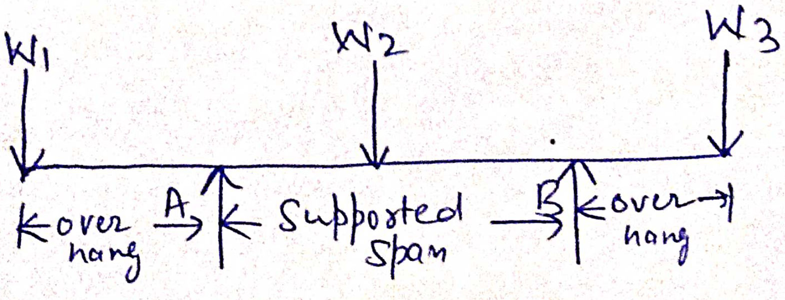
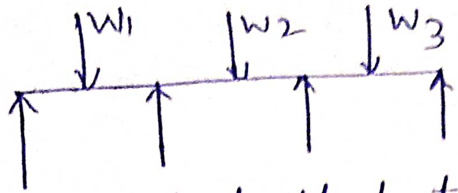


Fig-4

over-hanging Beam

2.1.5 <u>Continous Beam</u> :- A beam is said to be continous beam if it is supported on more than two supports.

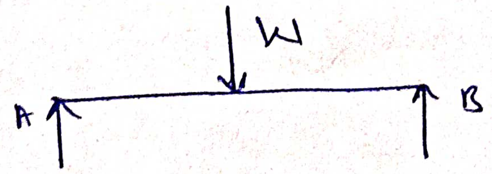


It may be noted that the ~~first~~ simply supported beams, cantilever and overhanging beams are statically determinate beams as their reactions at the support can be determined by the use of static equillibrium equations and are independent to the deformation of beams. Fixed beams and continous beams are statically indeterminate beams as their reactions cannot be calculated by the use of static equillibrium equations.

3. <u>Types of loading</u> :- Loading can be classified according to the application of load i.e point or concentrated load, uniformely distributed load (U.d.l), varying load.

3.1 <u>Point Load or Concentrated Load</u> :-

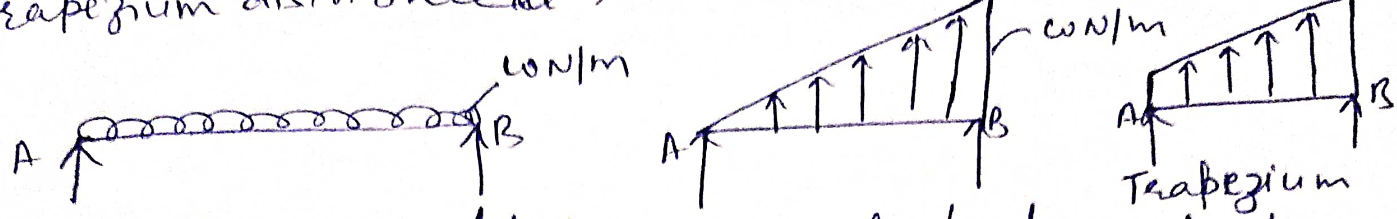
A load is said to be point load or concentrated load which is considered to ~~be~~ act at a point. In actual practice, the load has ~~to~~ be distributed over a small area because such small knife-edge contacts are generally neither possible nor desirable.



Point load

3.2 ~~Uniformly~~ <u>Distributed load</u> :- When the load is spread uniformly over the length of the beam i.e (at uniform rate say w kN/m run) it is said to be uniformly distributed load and is abbreviated as (U.d.L).

If the spread is not a uniform rate, it is said to be non-uniformly distributed load. Triangular and Trapezium distributed loads fall under this category.



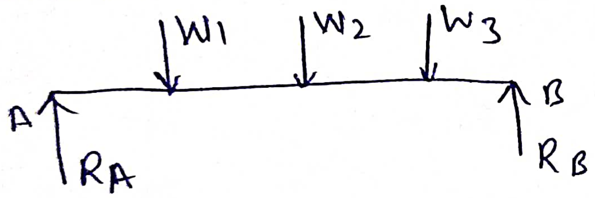
uniformly distributed load — Triangular load — Trapezium load

Note:- The total force due to u.d.l is (intensity of u.d.l × span) and line of action of this force passes through the mid of span for which it is spread.

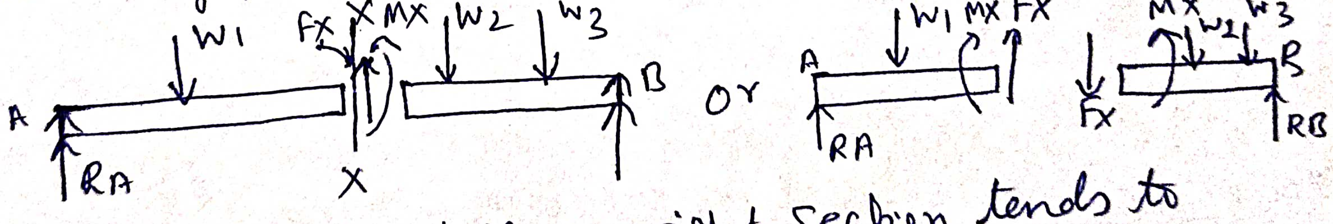
4.1 SHEAR FORCE (S.F.) AND BENDING MOMENT (B.M.)

Shearing force at any section of a beam represents the tendency for the portion of beam to one side of the section to slide or shear laterally relative to the other portion.

Consider a beam with points loads W_1, W_2 and W_3.



Now imagine that the beam is divided or sectioned by a sectioning plane X X as shown in fig



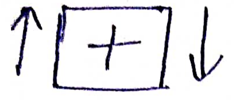
If either section i.e Left or right section tends to remain in equillibrium position a shear force Fx and bending moment Mx will act at section XX. So shear force may be defined as :- It is the ~~sum~~ algebric sum of all the vertical forces on either side of the section i.e either in left side of sectioning or right side of sectioning.

Similarly bending moment is defined as :- It is the

• algebric some of moments due to all the vertical forces in either direction of sectioning i.e either leftside or right side of the sectioning.

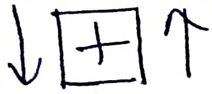
5. SIGN CONVENTION FOR SHEAR FORCE & BENDING MOMENT

5.1 SHEAR FORCE :-



SF

According to this sign convention the ~~forces in the~~ upward forces in the left side of sectioning ~~are taken~~ ~~as positive~~ and downward forces in right side of sectioning are taken as positive otherwise negative.

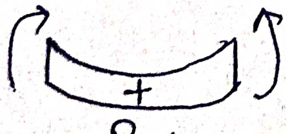


SF

According to this sign convention the downward forces in left and upward forces in the right side of the section are taken as positive otherwise negative.

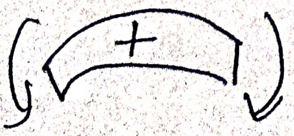
Note :- From the above two sign convention anyone can be assumed freely.

5.2 BENDING MOMENT



BM

According to this clockwise moment to the left of section and anticlockwise to the right of section are taken as positive. It is called sagging others are taken negative.



BM

Hogging

According to this anticlockwise moment in the left side and clockwise in the right side of the section are assumed positive others are taken negative. It is called Hogging.

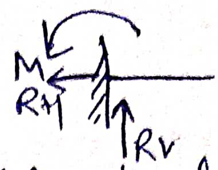
NOTE :- From the above two sign convention any one can be assumed freely.

6. Types of Supports

(a) Simply Support

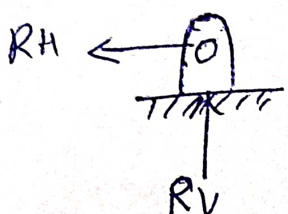
(only vertical Reaction)

(b) Fixed or built in support



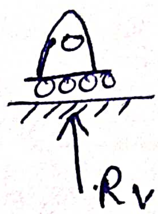
(A vertical reaction ~~and~~ fixing Moment ~~and~~ and a horizontal Reaction)

(c) Hinged support



(Vertical and Horizontal reaction)

(d) Roller Support

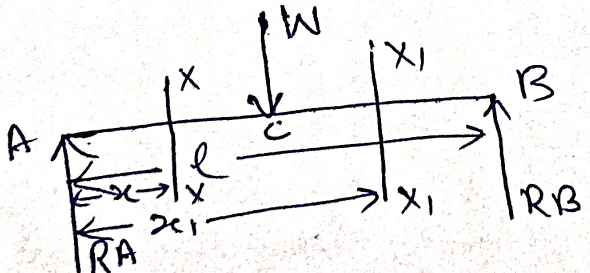


(Vertical Reaction only)

7. Simply Supported Beam with point load

7.1 Simply Supported beam with point load at Mid-span

Consider a beam of length l and a point load W acting at the mid span as shown in fig.



First calculate RA and RB the support reactions using static equillibrium equations

$$\Sigma F_y = 0$$

$$\Sigma M_y = 0$$

$$RA + RB - W = 0 \Rightarrow RA + RB = W \quad (1)$$

Taking Moment abt point B

$$RA \times l - W \times l/2 = 0 \Rightarrow RA = W/2$$

Put the value of RA in equation No-1

$RB = W/2$

Take section XX between A & C at a distance x from the left end A

SF at XX

$+RA = W/2$

SF in just left of point C

$= W/2$ (Remain constt. between A & C)

SF at X_1X_1

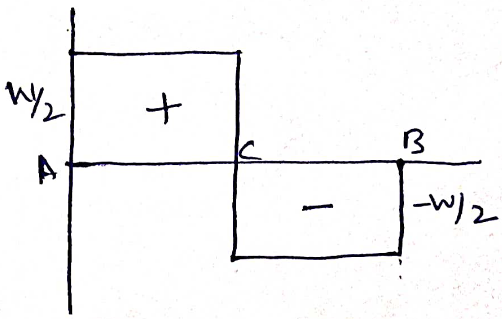
$= \frac{W}{2} - W = -W/2$ (Remain constt. between C and B)

BM at XX

$= RA \times x = +\frac{W}{2}x$ (Equation of linear line)

Bm at Point A

$= \frac{W}{2} \times 0 = 0$



BM in just left of point C

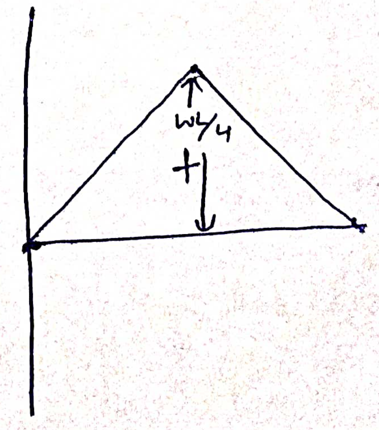
$= \frac{W}{2} \cdot \frac{l}{2} = \frac{Wl}{4}$

Bm at X_1X_1

$\frac{W}{2} \cdot x_1 - W(x_1 - l/2)$

Bm in just right of C

$\frac{W}{2} l/2 - W(l/2 - l/2) = \frac{WL}{4}$

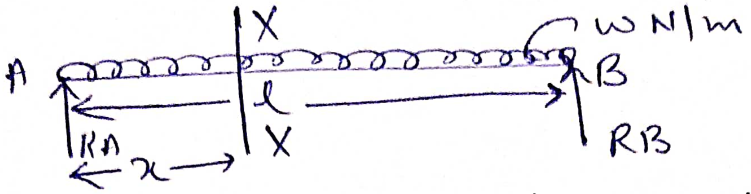


Bm at Point B

$\frac{W}{2} l/2 - W(l - l/2) = 0$

7.2 Simply supported beam with uniformely distributed load.

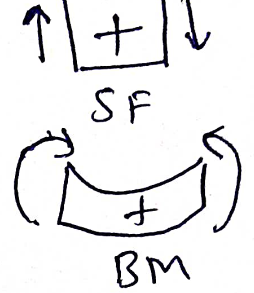
Let us consider a beam of length l and a u.d.l of intensity w N/m for its whole span.



First calculate support reactions at end supports A and B i.e RA & RB using static equillibrium equations.

$\Sigma f_y = 0$

$\Sigma M_y = 0$



$R_A + R_B = wl$ ——— ①

Taking moment about point B.

$R_A \times l = wl \cdot \frac{l}{2}$

$R_A = wl/2 \quad \therefore \quad R_B = \frac{wl}{2}$

Take Section XX at a distance x from the left end

SF at section XX

$= \frac{wl}{2} - wx$ (equation of linear line)

SF at point A

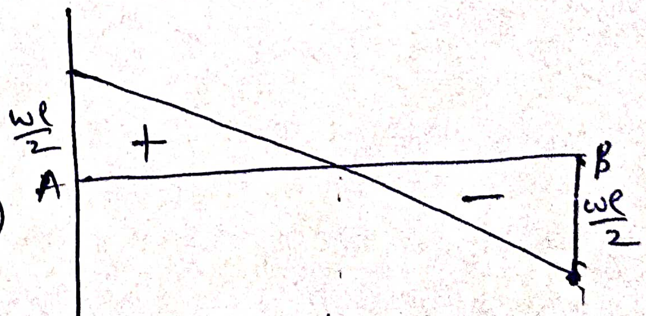
$= +\frac{wl}{2} \quad (\because x = 0)$

SF at Mid point $(x = l/2)$

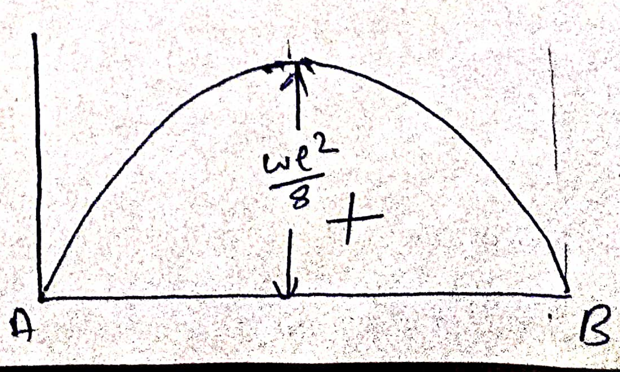
$= \frac{wl}{2} - \frac{wl}{2} = 0$

SF at Point B $(x = l)$

$= \frac{wl}{2} - wl = -\frac{wl}{2}$



SF diagram



B.M. at section XX

$$= \frac{wl}{2}x - wx \cdot \frac{x}{2} \Rightarrow \frac{wl}{2}x - \frac{wx^2}{2} \text{ (Quadratic Parabola)}$$

BM at point A $(x=0)$

$$= \frac{wl}{2}(0) - \frac{w(0)^2}{2} = 0$$

BM at mid point $(x = l/2)$

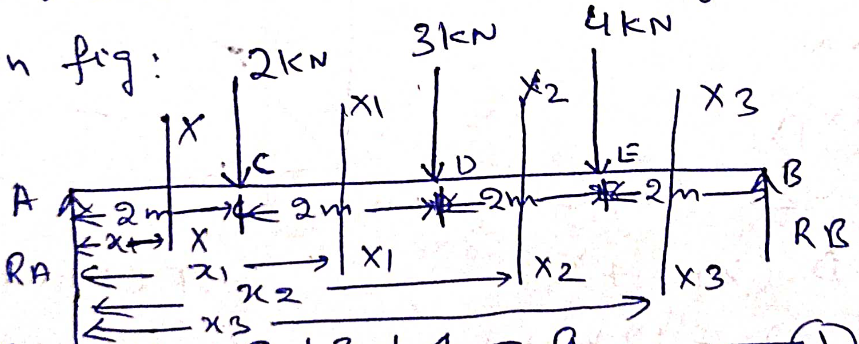
$$= \frac{wl}{2} \cdot \frac{l}{2} - \frac{w}{2}\left(\frac{l}{2}\right)^2 = \frac{wl^2}{8}$$

BM at point B $(x = l)$

$$= \frac{wl}{2} \cdot l - \frac{w}{2}(l)^2 = 0$$

Solved Examples

Problem 1. Draw S.F and B.M diagram for a beam show in fig:

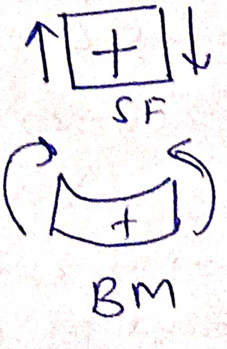


$RA + RB = 2 + 3 + 4 = 9$ —— (1)

Taking moment about point B

$RA \times 8 = 2 \times 6 + 3 \times 4 + 4 \times 2$

$RA = 32/8 = 4\,KN$



from equation I

$RB = 5\,KN$

P.T.O

SF at XX (i.e between A and C)
$= +4$ kN (Constant up to just left of point C from point A)

SF at X_1X_1 (i.e between C & D)
$= 4-2 = +2$ kN (Constant)

SF at X_2X_2 (i.e between D & E)
$= 4-2-3 = -1$ kN (Constant)

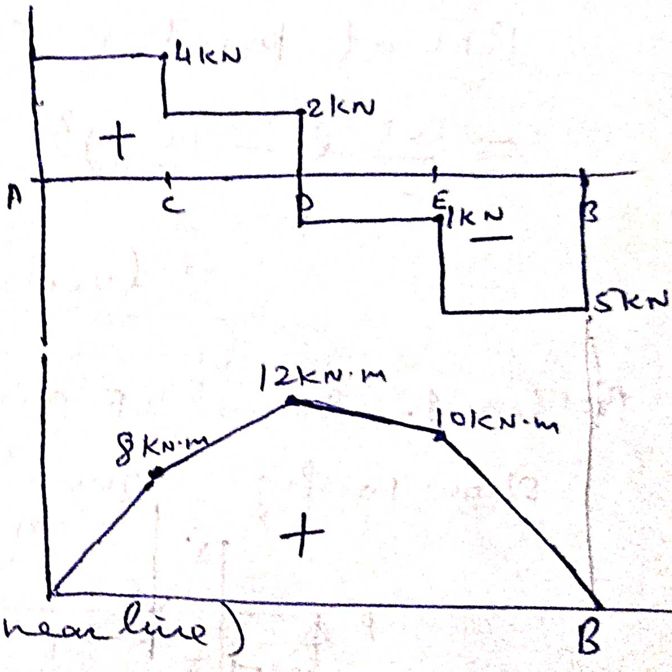
SF at X_3X_3 (i.e between E and B)
$= 4-2-3-4 = -5$ kN (Constant)

BM at XX
$= 4x$ (equation of Linear line)

BM at Point A $(x=0)$
$= 4(0) = 0$ kN·m

BM at point C
$= 4\times 2 = 8$ kN

BM at section X_1X_1
$= 4x_1 - 2(x_1-2)$ (equation linear line)



SF & BM diagram.

BM at point D $(x_1 = 4)$
$= 4\times 4 - 2(4-2) = 12$ kN·m

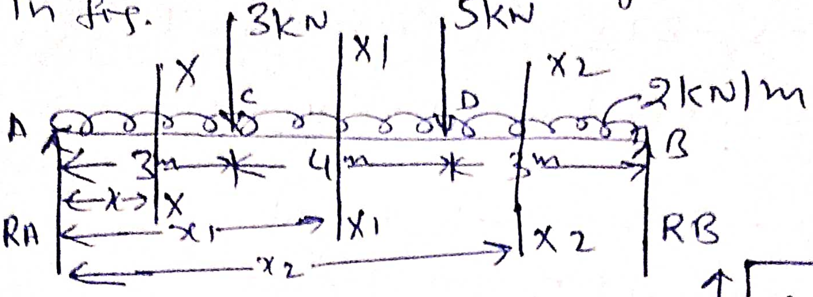
BM at ~~point~~ section X_2X_2
$= 4x_2 - 2(x_2-2) - 3(x_2-4)$ (Equation of linear line)

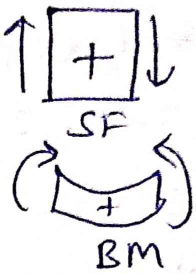
BM at point E $(x_2=6)$
$= 4\times 6 - 2(6-2) - 3(6-4) = 10$ kN·m

BM at section X_3X_3
$= 4x_3 - 2(x_3-2) - 3(x_3-4) - 4(x_3-6)$ (equation of linear line)

BM at point B $(x_3=8)$
$= 4\times 8 - 2(8-2) - 3(8-4) - 4(8-6) = 0$ kN·m

Problem :- 2 Draw the SF and BM diagram for the beam shown in fig.





First calculate Reactions at supports RA and RB.

$RA + RB = 3 + 5 + 2 \times 10 = 28 \text{ kN}$

Taking moment about point B

$RA \times 10 = 3 \times 7 + 5 \times 3 + 2 \times 10 \times \frac{10}{2}$

$RA = 136/10 = 13.6 \text{ kN}$

$\therefore RB = 14.4 \text{ kN}$

Take a section XX at a distance x from point A between A and C.

Now SF at section XX

$= 13.6 - 2x$ (equation of linear line)

SF at point A $(x = 0)$

$= 13.6 \text{ kN}$

SF in just left of point C $(x = 3)$

$= 13.6 - 2 \times 3 = 7.6 \text{ kN}$

Now Take section X_1X_1 between C and D at a distance x_1 from A.

$= 13.6 - 3 - 2x_1$ (Equation of linear line)

SF in just right of point C $(x_1 = 3)$

$= 13.6 - 3 - 2 \times 3 = 4.6 \text{ kN}$

SF in just left of point D $(x_1 = 7)$

$= 13.6 - 3 - 2 \times 7 = -3.4 \text{ kN}$

Again Take section X_2X_2 between D & B at a distance x_2 from point A.

SF at Section X_2X_2

$= 13.6 - 3 - 5 - 2x_2$ (equation of linear line)

SF in just right of point D ($x_2 = 7$)

$= 13.6 - 3 - 5 - 2\times 7 = -8.4$ kN

SF ~~is equal~~ at point B.

$= 13.6 - 3 - 5 - 2\times 10 = -14.4$ kN

<u>BM Calculations</u>

BM at section XX

$13.6x - 2\cdot x \cdot \frac{x}{2}$ (curve will be a parabola because it is quadratic equation)

BM at point A ($x=0$)

$= 13.6(0) - (0)^2 = 0$ kNm

BM at point C

$= 13.6\times 3 - (3)^2 = 31.8$ kN.m

BM at section X_1X_1

$13.6x_1 - 3(x_1 - 3) - 2\cdot x_1 \cdot \frac{x_1}{2}$

BM at ~~Section~~ point D ($x_1 = 7$)

$13.6\times 7 - 3(7-3) - (7)^2 = 34.2$ kN.m

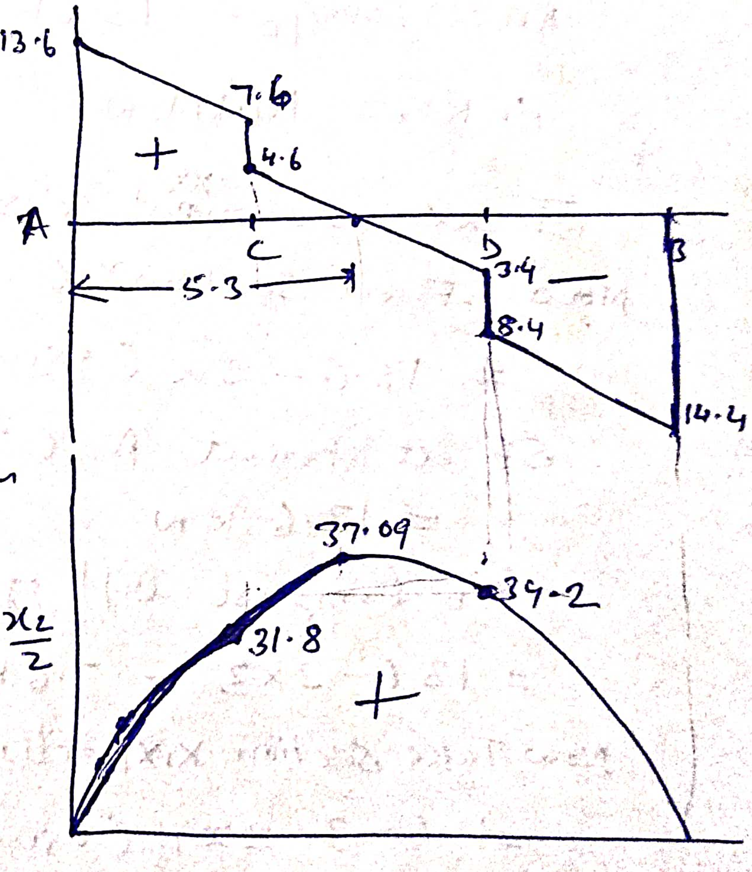
BM at section X_2X_2

$13.6\cdot x_2 - 3(x_2 - 3) - 5(x_2 - 7) - 2\cdot x_2 \cdot \frac{x_2}{2}$

BM at point B.

$13.6\times 10 - 3(10-3) - 5(10-7) - (10)^2$

$= 0$ kN.m



SF & BM diagram.

<u>Calculation of Max. Bending Moment</u>

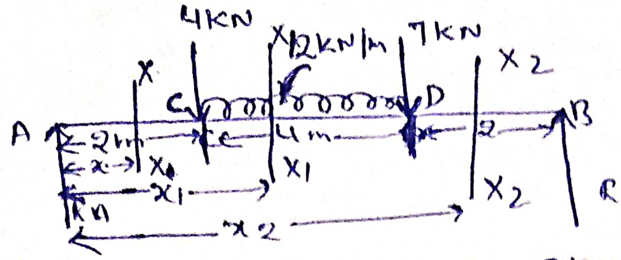
BM is max. where shear force is zero. SF is zero between C & D.

$\therefore \quad 13.6 - 3 - 2x_1 = 0 \Rightarrow x_1 = 5.3$ m

Put $x_1 = 5.3$ in BM equation at section X_1X_1

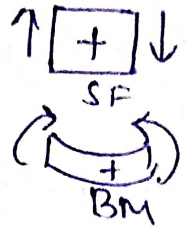
$= 13.6\times 5.3 - 3(5.3 - 3) - (5.3)^2 = 37.09$ kN.m (Max. BM)

Problem No. 3 Draw the SF and BM diagram for a beam shown in figure.



First calculate reactions at the supports i.e RA & RB

Sign convention



$RA + RB = 4+7+2\times4$ —(I)

Take moment about point B

$RA \times 8 = 4\times6 + 7\times2 + 2\times4\left(\frac{4}{2}+2\right)$ —(II)

$RA = 8.75$ KN

From (I)

$RB = 10.25$ KN

Take section between A & C at a distance x from left end A.

SF at Section XX

$+8.75$ KN (constt. between point A and C)

Take section X_1X_1 between C and D at a distance x_1 from left point A.

$8.75 - 4 - 2(x_1 - 2)$ [equation of linear line

SF in just right ~~left~~ of point C $(x_1 = 2)$

$= 8.75 - 4 = +4.75$ KN

SF in just left of point D $(x_1 = 6)$

$8.75 - 4 - 2(6-2) = -3.25$ KN

SF changes the sign between C&D

so SF is zero between C&D.

$8.75 - 4 - 2(x_1 - 2) = 0$

$4.75 + 4 = 2x_1 \Rightarrow x_1 = 4.375$ m

Take section X_2X_2 between D&B at a distance x_2 from point A.

$8.75 - 4 - 7 - 2\times4 = -10.25$ KN (constt)

P.T.O

BM calculations

BM at section XX

$8.75x$ (equation of linear line)

BM at point A $(x=0)$

$8.75 \times 0 = 0$

BM at point C $(x_1=2)$

$= 8.75 \times 2 = 17.5$ kN·m

BM at section X_1X_1

$8.75x_1 - 4(x_1-2) - 2(x_1-2)\cdot\frac{(x_1-2)}{2}$ (Quadratic equation)

BM at point D $(x_1 = 6)$

$8.75 \times 6 - 4(6-2) - (6-2)^2 = 20.5$ kN m

BM at section X_2X_2

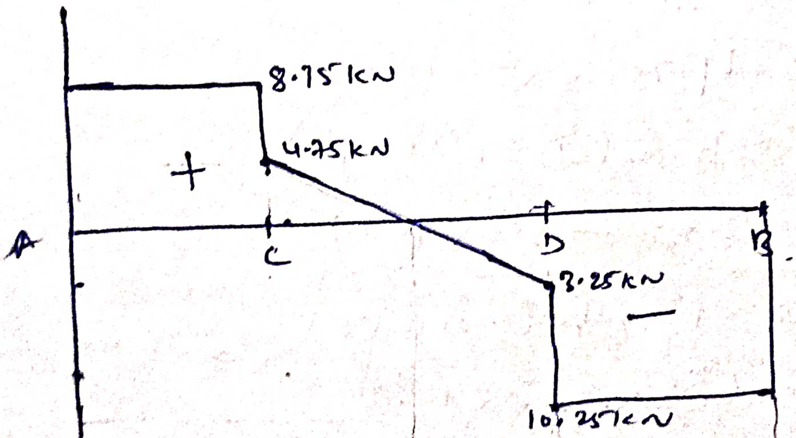
$8.75x_2 - 4(x_2-2) - 7(x_2-6) - 2\times4(x_2-2-\frac{4}{2})$

BM at point B $(x_2 = 8)$

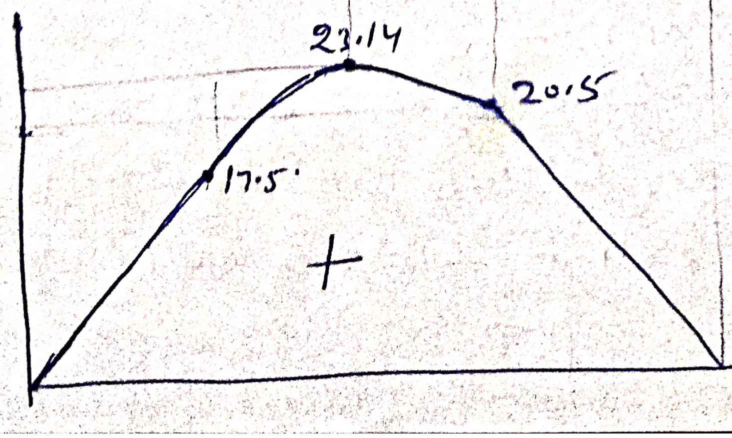
$8.75 \times 8 - 4(8-2) - 7(8-2) - 2\times4(8-4) = 0$

BM is max where SF is zero i.e $x_1 = 4.375$ m

$(BM)_{max} = 8.75 \times 4.375 - 4(4.375-2) - (4.375-2)^2 = 23.14$ kN m

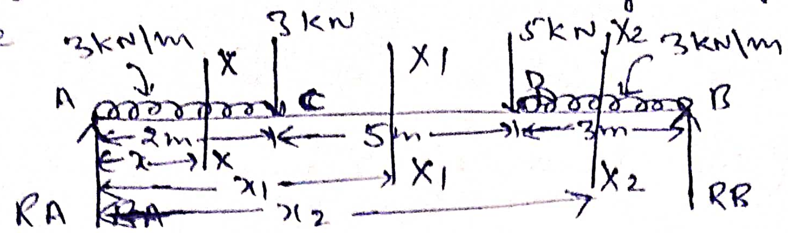


SF diagram



BM diagram

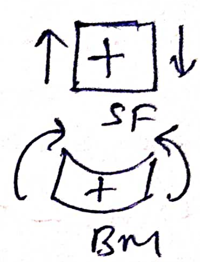
Problem 4 Draw the SF and BM diagram for the beam shown in figure



First calculate the reactions at the supports A and B

$$R_A + R_B = 3 + 5 + 3\times2 + 3\times3 = 23$$

Sign convention



Take moment about point B

$$R_A \times 10 = 3\times8 + 5\times3 + 3\times2\left(\frac{2}{2}+8\right) + 3\times3\times\frac{3}{2}$$

$$R_A = \frac{106.5}{10} = 10.65\ kN$$

$$R_B = 12.35\ kN$$

Take Section XX between A and C at a distance x from point A.

SF at Section XX

$10.65 - 3x$ (eqn. of linear line)

SF at point A $(x=0)$

10.65 KN

SF in just left of point C $(x=2)$

$10.65 - 3\times2 = 4.65$ KN

SF at section X_1X_1

$10.65 - 3 - 3\times2$ (Const from right of C to left of D)

$= 1.65$ kN

SF at Section X_2X_2

$10.65 - 3 - 5 - 3\times2 - 3(x_2 - 7)$ [eqn of linear line]

SF in just right of D $(x_2 = 7)$

$= -3.35$ kN

SF at point B $(x_2 = 10)$

$10.65 - 3 - 5 - 3\times2 - 3(10-7) = -12.35$ kN

BM Calculations

BM at section X X

$10.65x - 3 \cdot x \cdot \frac{x}{2}$ (curve between A & C remain parabola)

BM at point A ($x=0$)

$10.65(0) - \frac{3}{2}(0)^2 = 0$

BM at point C ($x=2$)

$10.65 \times 2 - \frac{3}{2}(2)^2 = 15.3$ kN.m

BM at Section X_1X_1

$10.65x_1 - 3(x_1-2) - 3\times2(x_1-\frac{2}{2})$ (Equation linear line)

BM at point D ($x_1 = 7$)

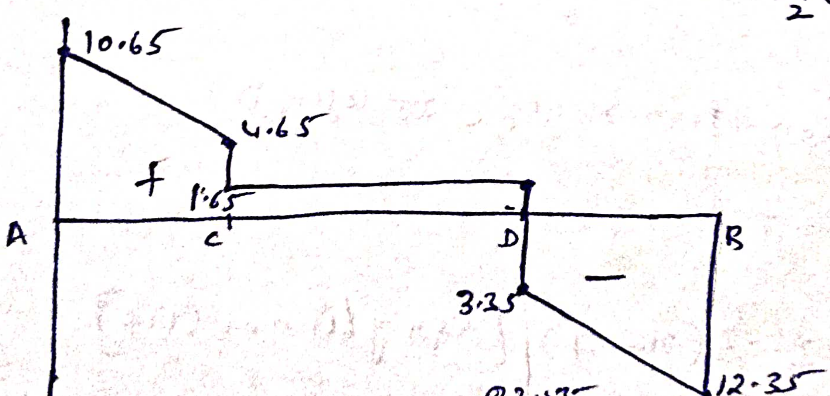
$10.65\times7 - 3(7-2) - 3\times2(7-1) = 23.55$ kN.m

BM at section X_2X_2

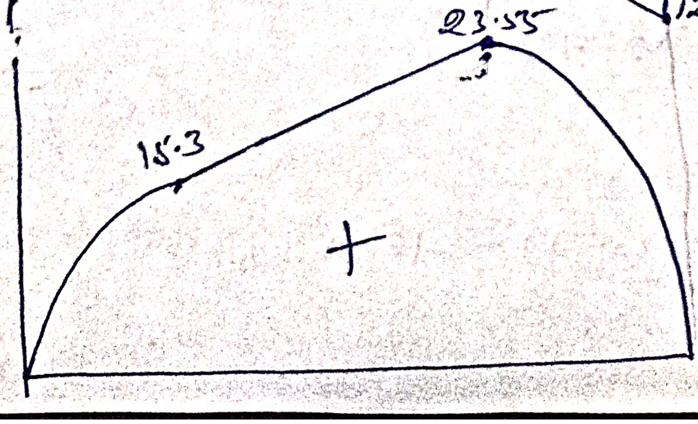
$10.65x_2 - 3(x_2-2) - 3\times2(x_2-\frac{2}{2}) - 5(x_2-7) - 3(x_2-7)\frac{(x_2-7)}{2}$
(curve remain parabola)

BM at point B.

$10.65\times10 - 3(10-2) - 3\times2(10-1) - 5(10-7) - \frac{3}{2}(10-7)^2 = 0$ kN.m

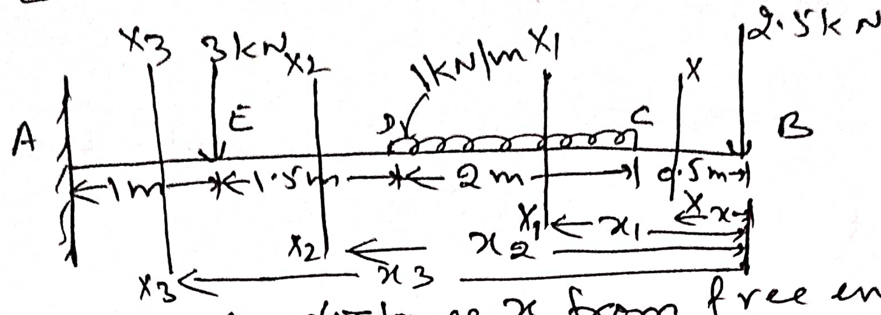


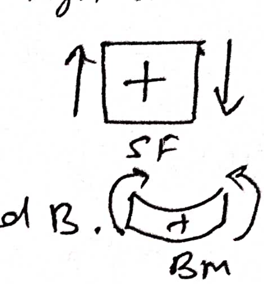
SF diagram



BM diagram

Problem: A cantilever of length 5.0 m is loaded as shown in fig. Draw the SF and BM diagrams for the cantilever.





Take section XX at a distance x from free end B.

SF at section XX

$= +2.5$ kN (Remains constt. between B & C)

Take section X_1X_1 at a distance x_1 from the free end B.

$2.5 + 1(x_1 - 0.5)$ [vary linearly between C and D]

SF in just left of point C ($x_1 = 0.5$)

$2.5 + 1(0.5 - 0.5) = 2.5$ kN

SF in just right of D ($x_1 = 2.5$)

$2.5 + 1(2.5 - 0.5) = 4.5$ kN

SF at section X_2X_2

$2.5 + 2 \times 1 = 4.5$ kN (Constt between D and E)

SF at section X_3X_3

$= 2.5 + 2 \times 1 + 3 = 7.5$ kN (Constt between E and A)

BM calculations

BM at section XX

$= -2.5x$ (vary linearly)

BM at point B ($x = 0$)

$= -2.5 \times 0 = 0$ kN.m

BM at point C ($x = 0.5m$)

$-2.5 \times 0.5 = -1.25 kN.m$

BM at section X_1X_1

$= -2.5x_1 - 1(x_1-0.5)\frac{(x_1-0.5)}{2}$ (Parabolic between C&D)

BM at point D ($x = 2.5$)

$= -2.5 \times 2.5 - \frac{(2.5-0.5)^2}{2} = -8.25 kN.m$

BM at section X_2X_2

$-2.5x_2 - 1 \times 2(x_2 - 1.5)$ [vary linearly between D&E

BM at point E ($x_2 = 4$)

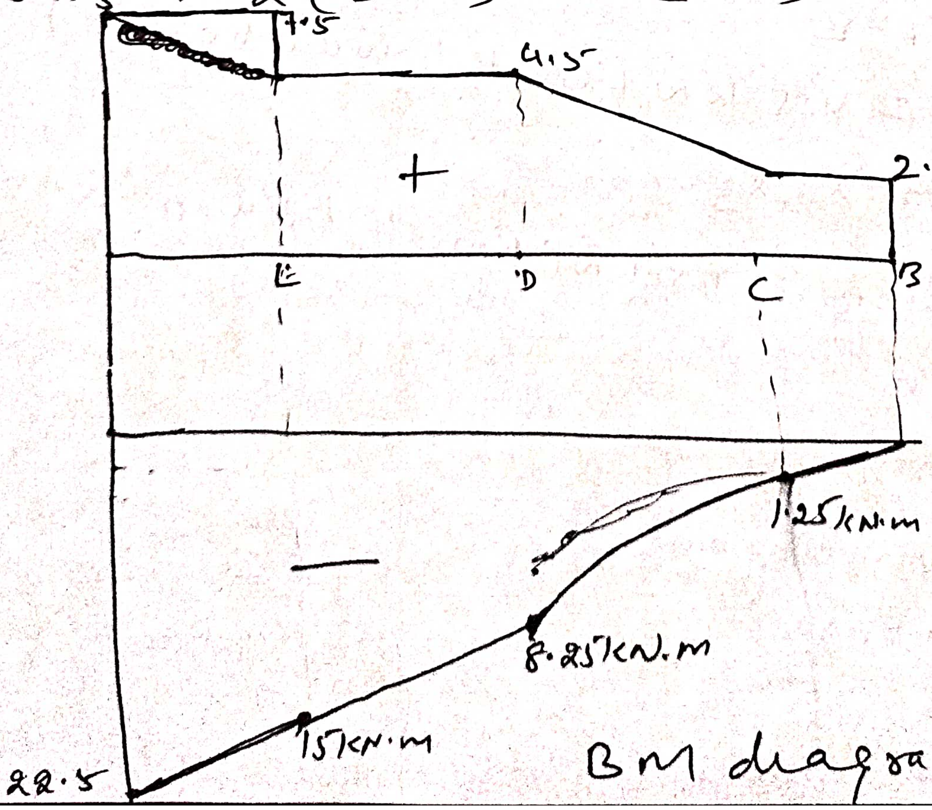
$-2.5 \times 4 - 2(4-1.5) = 15 kN.m$

BM at section X_3X_3

$-2.5x_3 - 1 \times 2(x_3-1.5) - 3(x_3-4)$ [vary linearly between E&A]

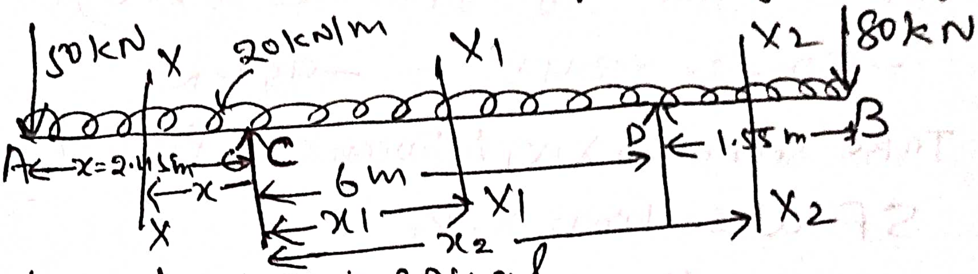
BM at point A ($x_3 = 5$)

$-2.5 \times 5 - 2(5-1.5) - 3(5-4) = 22.5 kN.m$



BM diagram

Problem A beam AB 10 metres long carries a uniformly distributed load of 20 kN/m over its entire length together with concentrated load of 50 kN at the left end A and 80 kN at the end B. The beam is to be supported at two props at the same level, 6 metres apart so that the reaction is the same at each. Determine the positions of the supports and draw SF and BM diagrams. Find the value of Maximum BM. Locate the points of contraflexure if any.



Since two support reactions are equal

$\therefore \quad R_C = R_D$

But $\quad R_C + R_D = 50 + 20 \times 10 + 80 = 330 \text{ kN}$

$\therefore R_C = R_D = 165 \text{ kN}$

Assume the overhang on the left side to be x metres, the overhang then, on the right side is $(10 - 6 - x) = (4 - x)$ m

To determine the value of x, taking moments about A, we get

$$R_C x + R_D (6 + x) = 20 \times 10 \times \frac{10}{2} + 80 \times 10$$

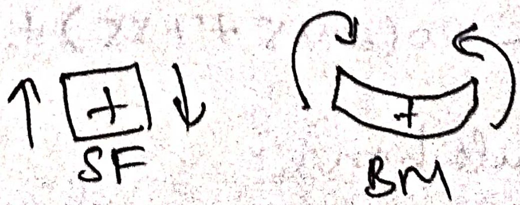
or $165x + 165(6 + x) = 1000 + 800$

or $165x + 990 + 165x = 1800$

or $x = 2.45 \text{ m} \quad \therefore$ overhang $AC = 2.45$ m

overhang $BD = 4 - 2.45 = 1.55$ m

Sign convention



P.T.O

Take a section in the left side of support C at a distance x from the support C.

SF at section XX

$-50-20(2.45-x)$ [equation of linear line between C&A]

SF at point A

$-50-20(2.45-2.45)=-50\,kN$

SF in just left of point C $(x=0)$

$-50-20\times2.45=-99\,kN$

Take section X_1X_1 between C&D from point C at a distance x_1

SF at section X_1X_1

$-50+165-20(2.45+x_1)$ [eqn of linear line]

SF in just right of point C $(x_1=0)$

$-50+165-20(2.45+0)=+66\,kN$

SF at just left of point D $(x_1=6)$

$-50+165-20(2.45+6)=-54\,kN$

Take section X_2X_2 between D&B at a distance x_2 from point C.

SF at section X_2X_2

$-50+165-20(2.45+x_2)+165$ [Equation of linear line]

SF in just right of point D $(x_2=6)$

$-50+165-20(2.45+6)+165=+111\,kN$

SF at point B $(x_2=7.55)$

$-50+165-20(2.45+7.55)+165=+80\,kN$

<u>BM calculations :</u>

B.M at Section XX

$$-50(2.45-x)-20\frac{(2.45-x)^2}{2}$$ [Quadratic eqn hence curve of BM will be parabola]

BM at point A $(x=2.45)$

$$-50(2.45-2.45)-20\frac{(2.45-2.45)^2}{2}=0\,kN.m$$

BM at point C $(x=0)$

$$-50(2.45-0)-20\frac{(2.45-0)^2}{2}=-182.52\,kN.m$$

BM at Section X_1X_1

$$-50(2.45+x_1)+165x_1-20\frac{(2.45+x_1)^2}{2}$$ (parabola)

BM at point D $(x_1=6)$

$$-50(2.45+6)+165\times6-20\frac{(2.45+6)^2}{2}=-148.02\,kN.m$$

BM at Section X_2X_2

$$-50(2.45+x_2)+165\times x_2-20\frac{(2.45+x_2)^2}{2}+165(x_2-6)$$

BM at point B at $(x_2=7.55)$

$$-50(2.45+7.55)+165\times7.55-20\frac{(2.45+7.55)^2}{2}+165(7.55-6)$$
$$=0\,kN.m$$

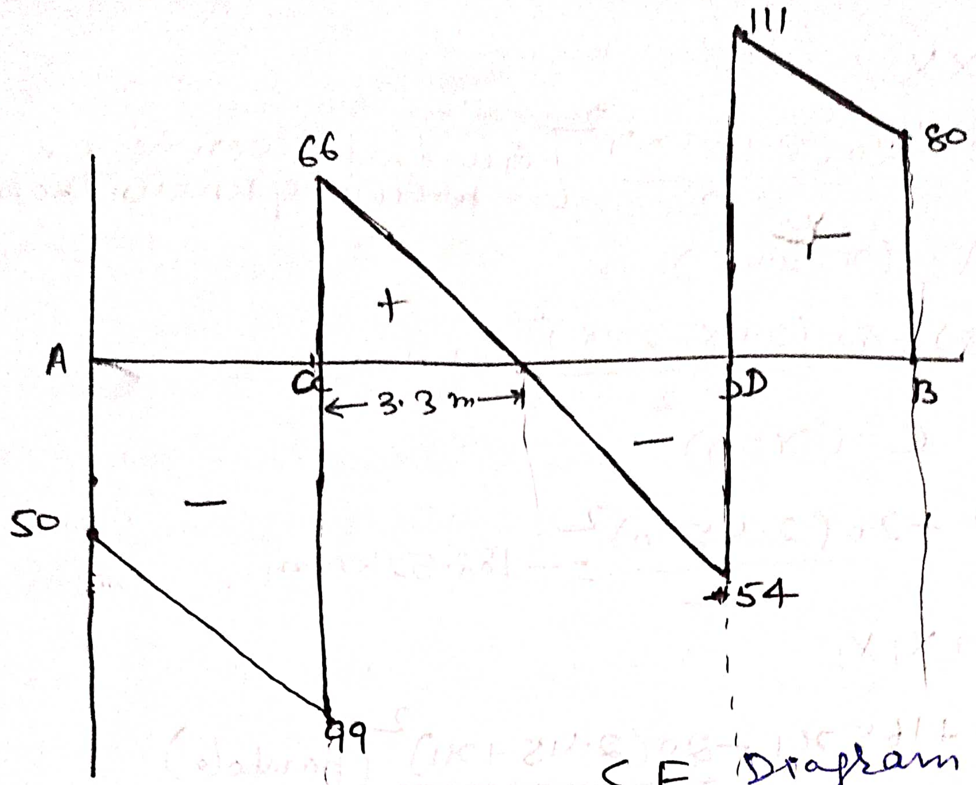
SF is zero between C&D so Put eqn of SF at $X_1X_1=0$

$$-50+165-20(2.45+x_1)=0$$
$$\therefore x_1=3.3\,m$$

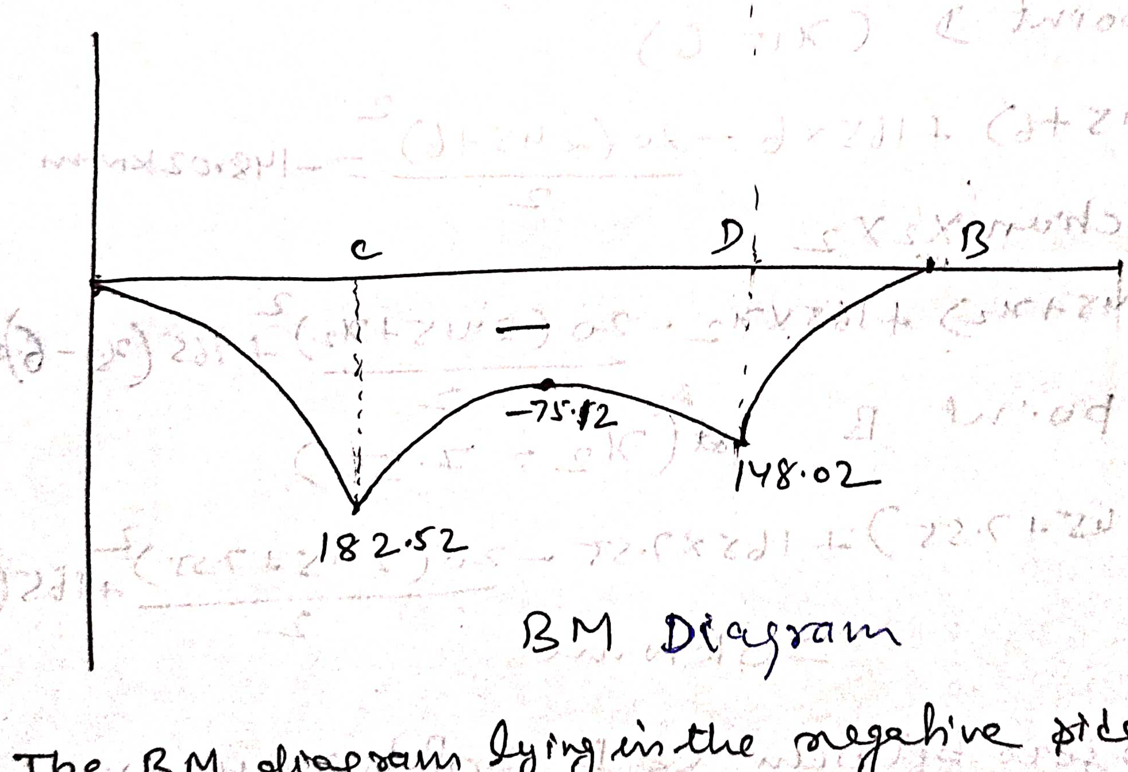
BM at $x_1=3.3\,m$

$$-50(2.45+3.3)+165\times3.3-20\frac{(2.45+3.3)^2}{2}=-75.12\,kN.m$$

P.T.O



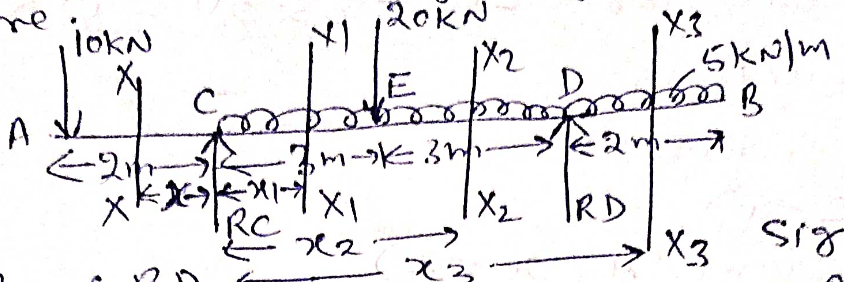
SF Diagram

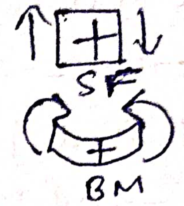


BM Diagram

The BM diagram lying in the negative side, it is not changing its sign so there is no point of contraflexure.

Problem Draw the SF and BM diagram for a overhang beam as shown in figure





Calculate Rc & RD

$R_C + R_D = 10 + 20 + 5\times 8 = 70\,kN$

Take moment about point D

$R_C \times 6 = 10\times 8 + 20\times 3 + 5\times 6\times \frac{6}{2} - 5\times 2\times \frac{2}{2} = 220$

$R_C = 36.66\,kN$, $R_D = 33.34\,kN$

Take section XX at a distance x in the left side of point C.

SF at section XX

$-10\,kN$ (constt between A and C)

SF at section X_1X_1

$-10 + 36.66 - 5x_1$ [eqn of linear line]

SF at just right of C ($x_1=0$)

$-10 + 36.66 - 5\times 0 = 26.66\,kN$

SF at just left of E ($x_1=3$)

$-10 + 36.66 - 5\times 3 = 11.66\,kN$

SF at section X_2X_2

$-10 + 36.66 - 5x_2 - 20$ (equation of linear line)

SF in just right of E ($x_2=3$)

$-10 + 36.66 - 5\times 3 - 20 = -8.34\,kN$

SF in just left of D ($x_2=6$)

$-10 + 36.66 - 5\times 6 - 20 = -23.34\,kN$

SF at section X_3X_3

$-10 + 36.66 - 5x_3 - 20 + 33.34$ (Equation of linear line)

SF in just right of D ($x_3=6$)

$-10 + 36.66 - 5\times 6 - 20 + 33.34 = +10\,kN$

P.T.O

SF at point B

$-10 + 36.66 - 5 \times 8 - 20 + 33.34 = 0\,kN$

BM calculations

BM at section XX

$-10(2-x)$

BM at point A ($x = 2$)

$-10(2-2) = 0\ kN\cdot m$

BM at point C

$-10(2-0) = -20\,kN\cdot m$

BM at section X_1X_1

$-10(2+x_1) + 36.66\,x_1 - 5 \cdot x_1 \cdot \frac{x_1}{2}$ (Parabolic curve between C & E)

BM at point E ($x_1 = 3$)

$-10(2+3) + 36.66 \times 3 - \frac{5}{2}(3)^2 = 37.48\,kN\cdot m$

BM at section X_2X_2

$-10(2+x_2) + 36.66\,x_2 - 5 \cdot x_2 \cdot \frac{x_2}{2} - 20(x_2 - 3)$ (Parabolic curve)

BM at point D ($x_2 = 6$)

$-10(2+6) + 36.66 \times 6 - \frac{5}{2}(6)^2 - 20(6-3) = -10.04\,kN\cdot m$

BM at section X_3X_3

$-10(2+x_3) + 36.66\,x_3 - 5 \cdot x_3 \cdot \frac{x_3}{2} - 20(x_3 - 3) + 33.34(x_3 - 6)$

(Parabolic curve

BM at point B ($x_3 = 8$)

$-10(2+8) + 36.66 \times 8 - \frac{5}{2}(8)^2 - 20(8-3) + 33.34(8-6) = 0\,kN\cdot m$

BM changes its sign between E & D i.e point of contraflexure. and also between C & E.

$-10(2+x_2) + 36.66\,x_2 - \frac{5}{2}x_2^2 - 20(x_2 - 3) = 0$

$-20 - 10x_2 + 36.66x_2 - \frac{5}{2}x_2^2 - 20x_2 + 60 = 0$

$2.5\,x_2^2 - 6.66x_2 - 40 = 0$

$x_2^2 - 2.664\,x_2 - 16 = 0$

$$x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x_2 = \frac{-(-2.664) \pm \sqrt{(2.664)^2 - 4\times1\times(-16)}}{2\times1}$$

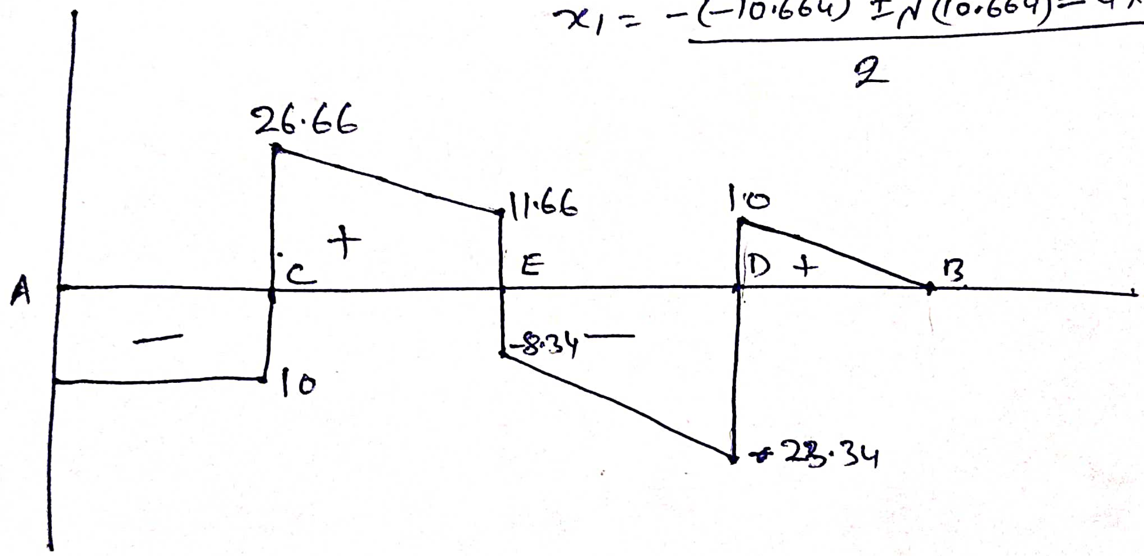
$$x_2 = \frac{2.664 \pm 8.43}{2} = 5.54\,m$$

Point of contraflexure lie at a distance of 5.54m and 0.81m from point C.

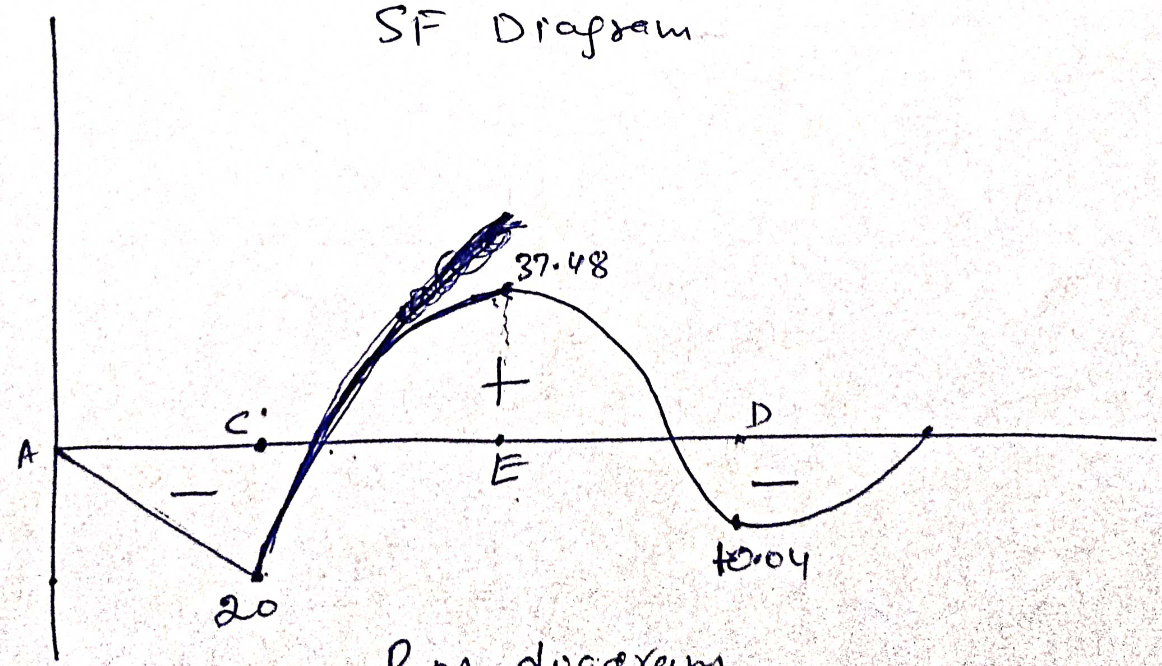
$$-20-10x_1 + 36.66x_1 - \frac{5}{2}x_1^2 = 0$$

$$x_1^2 - 10.664x_1 + 8 = 0$$

$$x_1 = \frac{-(-10.664) \pm \sqrt{(10.664)^2 - 4\times1\times8}}{2} = 0.81m$$



SF Diagram



BM diagram

Highlights :-

① Shear Force is the algeboric sum of all the vertical forces in either direction of the section i.e either in left side of the section or right of the section.

② Bending Moment is the algeboric sum of ~~all the~~ moments due to all the vertical forces in either direction of the section.

③ Any direction can be assumed for shearforce and Bending Moment.

④ Bending Moment is maximum where shear force is zero or vice-versa.

⑤ The point of zero bending moment is called point of contreflexture or inflexion. It occurs when Bending Moment change its sign from positive to negative or vice-versa.

⑥ The curve of shear force for point load is horizontal and contant along the length. For U.d.l it vary linearly.

⑦ The curve of BM diagram for point load vary linearly along the length while for u.d.l it is parabolic.

⑧ At simple supports and free ends B.M. is zero

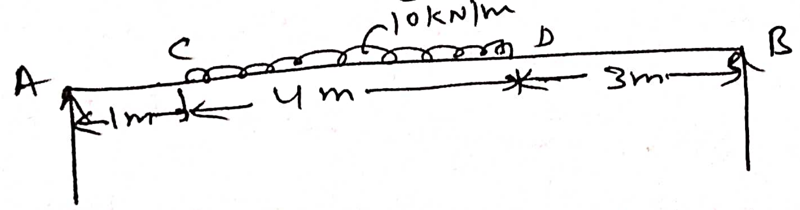
⑨ For u.d.l the line of action of the force passes through the mid-point of the beam span for which it is spread.

Problems

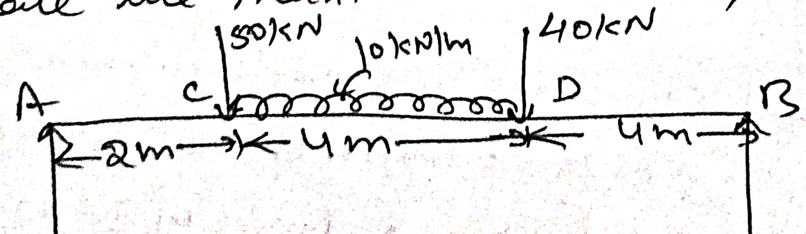
Q1. A simply supported beam of length 6 m, carries point load of 3 kN and 6 kN at distances of 2 m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam.

Q2 Draw the shear force and bending moment diagram for a simply supported beam of length 9 m and carrying a uniformly distributed load of 10 kN/m for a distance of 6 m from the left end. Also calculate the maximum bending moment at the section.

Q3 Draw the Shear force and B.M. diagrams for a simply supported beam of length 8 m and carrying a uniformly distributed load of 10 kN/m for a distance of 4 m as shown in the figure

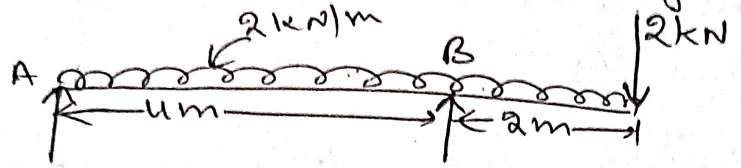


Q4 A simply supported beam of length 10 m, carries the uniformly distributed load and two points load as shown in the figure. Draw the SF and BM diagram for the beam. Also calculate the maximum bending moment.

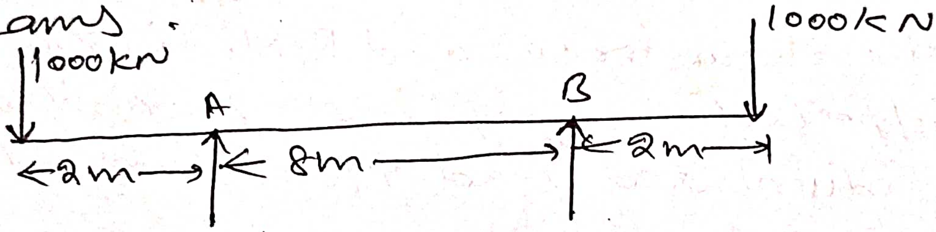


Q5 Draw the shear force and bending moment diagrams for the over-hanging beam carrying uniformly distributed load of 2 kN/m over the entire length. Also locate the point of contraflexure.

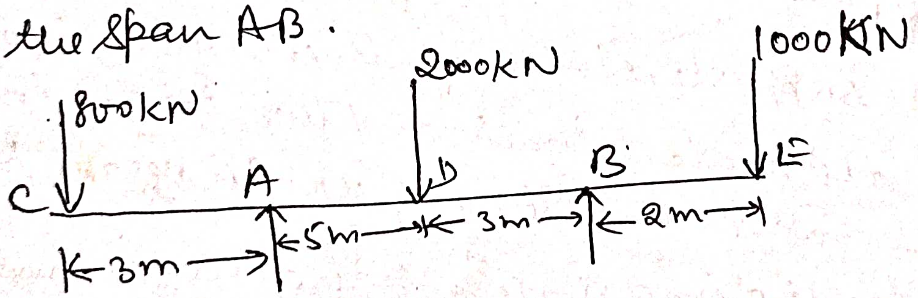
Q6 Draw the SF and BM diagrams for the over-hanging beam carrying uniformly distributed load of 2kN/m over the entire length and a point load of 2kN as shown in the figure. Locate the point of contraflexure.



Q7 A beam of length 12m is simply supported at two supports which are 8m apart, with an overhang of 2m on each side as shown in the figure. The beam carries a concentrated load of 1000kN at each end. Draw SF and BM diagrams.



Q8 Draw the SF and BM diagrams for the beam which is loaded as shown. Determine the points of contraflexure with in the span AB.



Q9 A cantilever 1.5m long is loaded with a uniformly distributed load of 2kN/m run over a length of 1.25m from the free end. It also carries a point load of 3kN at a distance of 0.25m from the free end. Draw the Shear force and Bending Moment diagrams of the cantilever.